

SUPPORTING SPECIAL NEEDS STUDENTS IN DRAWING MATHEMATICAL REPRESENTATIONS

Technology in Practice Strand

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1. ABSTRACT

The *INK-12: Interactive Ink Inscriptions in K-12* project, a collaboration between MIT and TERC, has been investigating the use of a pen-based wireless classroom interaction system in upper elementary math and science classes for the past four years [2]. This paper reports on a study that investigated the ways in which a pen-based drawing tool could support 4th and 5th grade special needs students in learning multiplicative reasoning. The drawing tool is what we call a *stamp*, which enables students to draw an image, then duplicate the image to create a mathematical representation, e.g., four groups of six. We worked with a class of eight special needs students for 10 classroom sessions of between 45 minutes and an hour, using a structured sequence of multiplication and division problems. We identified several specific ways in which stamps helped the students gain more proficiency with multiplicative reasoning and based on pre- and post- assessments, saw evidence of their academic progress.

2. PROBLEM STATEMENT AND CONTEXT

Asking elementary students to draw representations of mathematical situations is a common strategy for helping them understand the meaning of basic operations. Teachers often suggest that students “draw a picture” to help them figure out what a problem is asking. Mathematics education researchers have studied children’s drawings in response to math problems and see them as an important bridge between the specific and concrete objects in the problem—and the abstractions that underlie mathematical reasoning. Woleck [5], for example, notes that students often use drawings as mathematical manipulatives, similar to Unifix cubes or counters, allowing them to create a picture of a situation, then count the objects in it. She notes that if students need to draw multiple copies of the same object, they often begin to simplify the drawing, leaving out unnecessary features that might have been important in an artistic context, but become cumbersome in a mathematical context. Woleck also regards students’ drawings as “springboards for the talking of mathematics,” a way for students to share their mathematical thinking and their further questions.

Fosnot and Dolk [1] examine the role drawing can play in students’ learning of multiplication and division. In multiplication, students need to be able to count groups, each of which contains multiple objects. This need requires students to make a major shift in perspective from looking at individual objects to focusing on the group of objects as an individual entity, a process called *unitizing*. Fosnot and Dolk describe drawings—either presented by teachers or created by students—that support this unitizing process by showing objects grouped “naturally,” e.g., people sitting at tables of 6, games packed 10 to a box, or apples in boxes of 6. Similarly, Smith [4] describes how children drew multiple copies of an object (in this case, candy bars) to support their thinking about a division problem.

Drawing many identical images, however, can end up being a tedious distraction from the mathematical core of a problem, and students often make unintentional errors by drawing one too many or one too few copies. A pen-based system has just the right affordances to simplify the creation of multiple copies of a drawn image. The INK-12 project has been investigating the use of a pen-based classroom interaction system in upper elementary math and science classes for the past four years [2]. As part of this project, we designed and implemented “stamps,” digital structures that could be used to create multiple, identical “stamped” images that we hypothesized would support students’ learning of multiplication and division concepts. This paper reports on the use of the INK-12 technology in a special education class of 4th and 5th graders who were learning multiplicative reasoning, focusing on their use of stamps.

The research reported in this paper is part of a broader research question about ways in which digital tools can enhance students’ ability to express their mathematical and scientific understandings through drawing. By providing students with the ability to easily make multiple copies of an image, our software, which we call Classroom Learning Partner (CLP) adds structure to the inscriptions students create, without taking away the creativity and ownership freehand drawing confers. Another advantage of this added structure is that it aids automatic interpretation of student drawings, as described in the companion paper Koile et al. [3].

3. METHOD EMPLOYED

The design process for the stamps is described elsewhere [3]. Crucial to the results reported in this paper is an important distinction: In creating students’ notebooks, we had the option of either including pre-made stamps, as in Figures 1 and 2 below, or a blank stamp on which the student would draw. A single notebook page also could include a combination of pre-drawn and blank stamps.

The results reported here are from 10 sessions of between 45 minutes and an hour in a substantially separate class of eight 4th and 5th grade students with disabilities in a working-class suburb of Boston. Students are assigned to this class based on having IQs below 70 as determined by district/state tests, but no behavioral issues such as autism. The class included two students who had limited knowledge of English and others who had difficulties with focusing and attention. The teacher had been working with the students on their multiplication facts, but did not have the resources to work on building conceptual understanding of the operations. She welcomed the INK-12 project as a way to help her students develop mathematical understanding beyond memorizing the facts.

Pre- and post-assessments with multiplication and division items were administered, both orally and in written form, to all the students in this class. Items included multiplication and division computation, solving story problems, and problems involving visualization, e.g., If there are 3 tennis balls in 1 can, how many are there in 4 cans?

4. RESULTS AND EVALUATION

Comparison of pre- and post- test results, described in more detail at the end of this section, showed some improvement in students’ understanding of multiplication, which we considered significant considering both the limited amount of time we actually spent with the students and their relatively low math achievement levels. The bulk of this section illustrates the ways in which students used the two varieties of stamps to solve multiplication and division problems.

We describe the following ways in which the stamps contributed to students' ability to create representations, which in turn supported the development of their multiplicative reasoning.

1. Students with fine motor control issues were less frustrated because they only had to draw one image and could erase without leaving a mess.
2. Students perseverated less on details of the drawings, since they only had to draw once.
3. Stamps provided a pre-defined, limited space for students to draw in.
4. Students could first create, then move stamped images, organizing their representations as they went along.
5. Pre-drawn stamps modeled unitizing.
6. Students created unitized drawings on stamps, modeled on pre-made examples.
7. Drawing in a stamp was a clear first step for problem-solving, especially useful for these students who often struggled with how to start working on a problem.

In the first lesson, students used a pre-made stamp, with the object in the story already drawn. The first stamps students encountered explicitly modeled *unitizing* by including the group of objects to be counted on the stamp, as in Figures 1 and 2 below. The group consists of 4 cookies; it is further made into a single "object" by being on a single plate. Most of the students were easily able to create a well-organized picture of 6 plates of cookies, then use it to count the total number of cookies. Mickey,¹ whose work is shown in Figure 1, counts by 2's to solve the problem. Other students, such as Eliza, used the stamps and repeated addition to solve the problem, as shown in Figure 2.

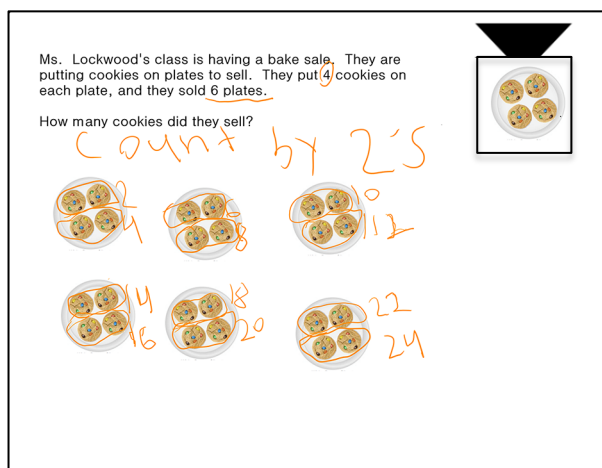


Figure 1. Mickey's work using stamps for 4 x 6

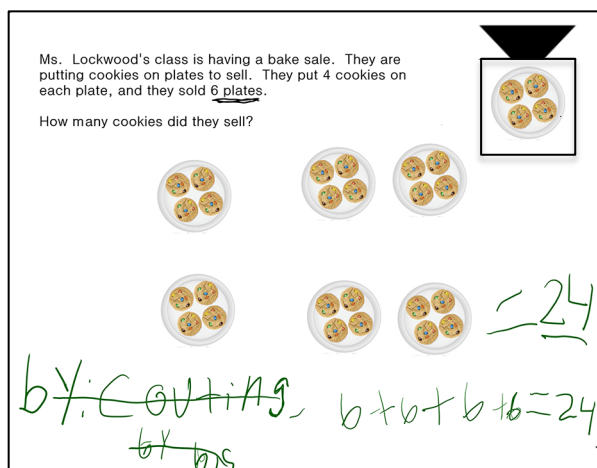


Figure 2. Eliza's work using stamps for 4 x 6

After students had some experience with pre-made stamps, problems were introduced in which students drew their own image on the stamp. The students were highly engaged by being able to make their own representations. The teacher reported that in addition to engaging the students, the stamps helped students organize their representations and kept them from perseverating over details of the drawing. She also noted that, while these students often had trouble deciding what to do when presented with a word problem, drawing their own stamp

¹ All student names are pseudonyms.

provided a useful first step. Not only did it give them something specific to do, but the act of drawing the stamp required them to think about the quantities in the problem in a way that seemed to prepare them for the next step—that of creating multiple stamped images.

The students with a limited knowledge of English did particularly well when they were able to draw their own stamps. Hector created a stamp of a car with four wheels, made several copies of it and used repeated addition to solve a multiplication problem about wheels, as shown in Figure 3. When he had been asked a similar wheel problem orally on the pre-test, he had not been able to solve it. Hector also used a combination of a pre-drawn stamp (the person) and one he drew himself (the pencil), as shown in Figure 4, to solve a problem where he had to group objects, rather than having them automatically “grouped” by virtue of being part of the same object, such as the car in Figure 3. Although he did not write the number sentence correctly (using the symbol for division instead of multiplication), he represented the problem correctly and knew what all of the parts meant.

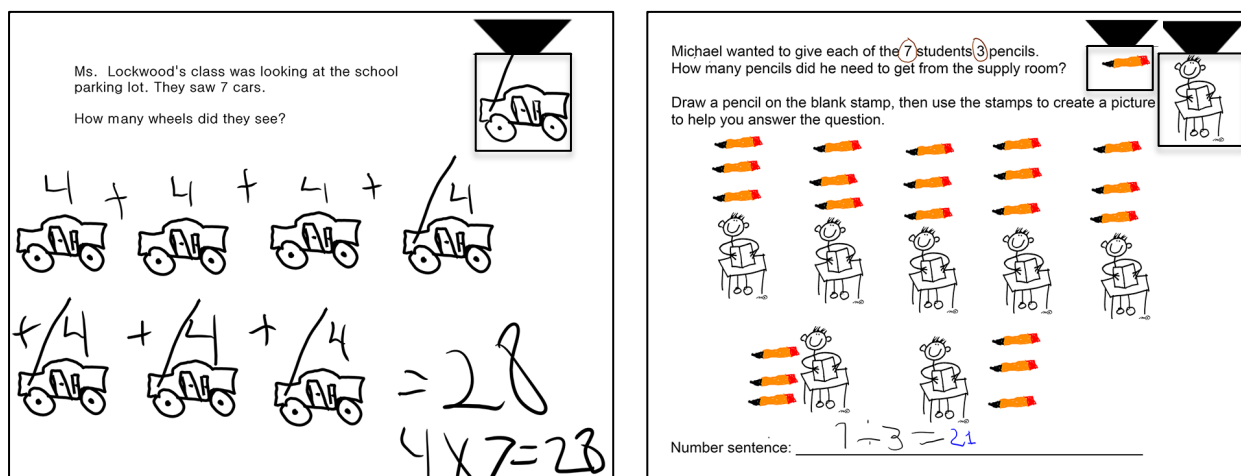



Figure 3. Hector’s work using a car stamp he drew Figure 4. Hector’s work using a pencil stamp he drew

Alfonso, another student with limited English proficiency, had more success drawing his own stamps, as opposed to using the pre-made stamps. He had some fine motor control difficulties with the pre-made stamps, which frustrated him, especially if he had to create a large number of stamped images. When he created his own stamp with 3 stickers on it for the problem: “If each student gets 3 stickers and there are 8 students, how many stickers are there in all?” he was able to represent and solve the problem correctly, as shown in Figure 5. Note that by creating a stamp with three stars on it, Alfonso seems to be showing some awareness of unitizing; he could have created a stamp with a single image on it (and some other students did). By creating a stamp with three stickers on it, he only had to create eight stamped images, rather than the 24 he would have had to create with a stamp of just one sticker. Alfonso also was able to use his drawings to correctly solve some division problems, as shown in Figure 6, even though when we started working with the class, he did not understand what division was and was unable to answer any division questions on the pre-assessment.


There are 8 people in the class.
Ms. Lockwood wants to give each person 3 star stickers.

How many stickers does she give out?

Use the blank stamp to create a picture to help you answer the question.



Star





Ms. Lockwood gives out 24 stickers in all.

Figure 5. Alfonso's work with a drawn stamp

Nael wants to play basketball. There are 25 students.
Each team has 5 students.

How many teams will there be?

Use the stamp to create a picture of the students. Draw a circle around each team of 5 students.

Number sentence: 25 ÷ 5 = 5


Figure 6. Alfonso's work on a division problem

For another student, Mickey, extended practice using stamps appeared to facilitate significant growth in his mathematical thinking. During one lesson using stamps, Mickey went from not representing the correct problem to being able to represent and solve a more complicated problem, as shown in Figures 7 and 8 below.


There are 8 people in our class.
Each person has 2 eyes.

How many eyes are there in all?

On the blank stamp draw a face with 2 eyes.
Then use the stamp to create a picture to help you answer the question.



Eyes




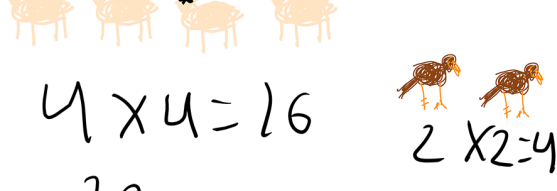
There are 24 eyes in all.

Figure 7. Mickey's incorrect work

Michael and Nael see 4 cats and 2 birds on the playground.

How many animal legs do they see in all?

Draw a cat stamp and a bird stamp. Then use the stamps to create a picture to help you answer the question.

They see 20 animal legs in all.

Figure 8. Mickey's correct work

The pre- and post-assessments suggest that the students from this special needs class made real progress in solving multiplication and division problems through their experience using CLP, especially the stamps. On the post-assessment, they were more confident in their ability to solve problems. They all had a strategy for each problem, even if it was counting by 1's, and were more likely to explain how they got an answer than they had been on the pre-assessment. For example, on the post-assessment:

1. For $20 \div 5$ one student said that she put cubes in her head, indicating that she put 20 in 5 groups of 4.
2. For 4×3 , one student said, "I know 3×3 is 9 and then he said 10,11,12."
3. One student solved $10 \div 2$ by going remembering that $5 + 5 = 10$ and reasoning that there were 2 groups of 5 in 10.
4. One student said " $36 \div 6$ is 6 because $6 \times 6 = 36$."

As there had been during the class sessions, there was considerable variability in students' ability to connect representations with number sentences on the post-assessment. Although a few students were able to connect representations with number sentences on the post-assessment, others were not. For example, for 7×3 , some students drew 7 things and 3 things, as opposed to 7 groups of 3, but had the correct number sentence. Despite this variability and especially given the difficulties these students often have with mathematics, we believe the affordances of the stamps are worth further exploration with this population.

5. FUTURE WORK

In order to help students move beyond the single-digit multiplication tables, we are planning to implement tools that enable students to draw and partition arrays. Array-based representations can be used to help students see how to think about a problem such as 24×36 as the sum of four simpler problems: 20×30 , 20×3 , 30×2 and 4×6 . We will be trying out these newer tools with non-special-education students in the coming year.

6. ACKNOWLEDGMENTS

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7. REFERENCES

- [1] Fosnot, C. T. and Dolk, M. *Young Mathematicians at Work: Constructing Multiplication and Division*. Portsmouth, N.H.: Heinemann, 2001.
- [2] Koile, K., Reider, D., and Rubin, A. INK-12: a pen-based wireless classroom interaction system for K-12. In *The Impact of Tablet PCs and Pen-based Technology on Education: Evidence and Outcomes*. Reed, R. and Berque, D. (editors). Purdue University Press, 2010.
- [3] Koile, K., and Rubin, A. Machine interpretation of students' hand-drawn mathematical representations. Submitted to WIPTTE, 2013.
- [4] Smith, S.P. Representation in school mathematics: Children's representations of problems. In Kilpatrick, J., Martin, W.G., & Schifter, Deborah (Eds.) *A Research Companion to Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2003, 263-274.
- [5] Woleck, K. R. Listen to their pictures: an investigation of children's mathematical drawings. In A. Cuoco (Ed.), *The roles of representation in school mathematics*. Reston, VA: National Council of Teachers of Mathematics, 2001, 215-227.